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Quantum noise in parametric amplification under phase-mismatched conditions

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ABSTRACT

This paper studies quantum noise in parametric amplification under phase-mismatched conditions. The equations of motion of the quantum-mechanical field operators, which include phase mismatch under unsaturated conditions are first derived from the Heisenberg equation. Next, the noise figure is evaluated using the solutions of the derived equations. The results indicate that phase mismatch scarcely affects noise property in phase-insensitive amplification while it has a notable effect in case of phase-sensitive amplification.

1. Introduction

When the signal light propagates through a nonlinear medium together with strong pump light(s), idler light is generated and the signal light is amplified, given that certain conditions are satisfied. This phenomenon is called parametric amplification. A feature of such amplification is low-noise property [1–6], which is beneficial for optical communication systems. Especially, phase-sensitive amplification (PSA), that is a particular type of parametric amplification in which the signal and idler lights are degenerate, offers a quantum-limited noise figure (NF) of 0 dB, which realizes noiseless amplification in principle.

In order to achieve signal amplification via parametric interaction, the propagation phases should be relatively matched among interacting lights, which is called the phase-matching condition. Thus, previous studies on quantum noise in parametric amplification assume that the phase-matching condition is satisfied. The above mentioned noiseless amplification in PSA is achievable under this condition as well. However, even when the phase-matching condition is not perfectly satisfied, signal amplification is still achievable. Nevertheless, there has been no study on quantum noise under such conditions, to the author's knowledge. Although Refs. [2–4], which refer to the quantum noise in parametric amplification, present the equations of motion of the quantum-mechanical field operators including the phase mismatch, the quantum noise property under phase-mismatched conditions is not investigated.

With the above background, this paper studies the dependence of phase mismatch on quantum noise in parametric amplification. Both phase-sensitive amplification (PSA) and phase-insensitive amplification (PIA) are treated. First, we derive the equations of motion of the field operators in parametric amplification using the Heisenberg equation, which explicitly include the phase mismatch. Next, using the solutions of the derived equations, the quantum-mechanical NF is evaluated. The results showed that NF scarcely depends on the phase mismatch in PIA, as compared with PSA, in which it is notably affected by the phase mismatch.

2. Theoretical treatment

2.1. Equation of motion

In quantum mechanics, a physical quantity is represented by an operator, and the behavior of a physical quantity operator follows the Heisenberg equation with the Hamiltonian of a concerned physical system. Regarding parametric amplification phenomena, Refs. [5,6] show the equation of motion of the annihilation operator (which corresponds to the optical field), derived from the Heisenberg equation, where the phase-matching condition is assumed to be satisfied. Refs. [2,3] show the equation including phase mismatch, but they are phenomenologically translated from classical nonlinear coupled-equations, not derived from the Heisenberg equation. Ref. [4] presents the equation for the field operator derived from the Heisenberg equation. Unfortunately, however, linear phase mismatch and self and cross phase modulations are separately treated, and thus the presented equation is not formed to straightforwardly investigate the dependence of quantum noise on the effective phase mismatch that consists of

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the linear phase mismatch and phase-shift due to self and cross phase modulations.

Therefore in this paper, we first derive the equations of motion of the field operators in parametric interaction systems including the effective phase mismatch. The Hamiltonian for a physical system, in which signal and idler photons are created from pump light(s), is expressed as [7]

$$\hat{H} = \hbar\omega_s\hat{a}_s^\dagger\hat{a}_s + \hbar\omega_i\hat{a}_i^\dagger\hat{a}_i + i\hbar(\chi\hat{a}_s^\dagger\hat{a}_i^\dagger - \chi^*\hat{a}_s\hat{a}_i), \tag{1}$$

where $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation (field) operator and its Hermitian conjugate (i.e., creation operator), respectively; ω is the light angular frequency; $\hbar$ is Planck's constant; χ is a proportional constant including the nonlinear coefficient and the classical pump light amplitude(s); and subscripts s and i indicate the signal and the idler, respectively. Using this Hamiltonian, the equations of motion for the signal annihilation operator $\hat{a}_s$ and the idler creation operator $\hat{a}_i^\dagger$ are derived from the Heisenberg equation as

$$\frac{d\hat{a}_s}{dt} = \frac{1}{i\hbar}[\hat{a}_s, \hat{H}] = -i\omega_s\hat{a}_s + \chi\hat{a}_i^\dagger, \tag{2a}$$

$$\frac{d\hat{a}_i^\dagger}{dt} = \frac{1}{i\hbar}[\hat{a}_i^\dagger, \hat{H}] = i\omega_i\hat{a}_i^\dagger + \chi^*\hat{a}_s. \tag{2b}$$

Eqs. (2a) and (2b) describe the time-evolution of the operators. They can be rewritten to describe the space-evolution during propagation along a nonlinear medium, by using the relationship of time t and spatial coordinate z (i.e., $z=(c/n)t$ where c is the light velocity in the vacuum and n is the refractive index), as

$$\frac{d\hat{a}_s}{dz} = -i\beta_s\hat{a}_s + \frac{n\chi}{c}\hat{a}_i^\dagger, \tag{3a}$$

$$\frac{d\hat{a}_i^\dagger}{dz} = i\beta_i\hat{a}_i^\dagger + \frac{n\chi^*}{c}\hat{a}_s, \tag{3b}$$

where $\beta=(n/c)\omega$ is the propagation constant in the medium. From these equations, we obtain the following equations by expressing $\hat{a}_s$ and $\hat{a}_i^\dagger$ as $\hat{a}_s(z) = \hat{b}_s(z)\exp(-i\beta_s z)$ and $\hat{a}_i^\dagger(z) = \hat{b}_i^\dagger(z)\exp(i\beta_i z)$, respectively.

$$\frac{d\hat{b}_s}{dz} = \frac{n\chi}{c}\hat{b}_i^\dagger e^{i(\beta_s+\beta_i)z}, \tag{4a}$$

$$\frac{d\hat{b}_i^\dagger}{dz} = \frac{n\chi^*}{c}\hat{b}_s e^{-i(\beta_s+\beta_i)z}. \tag{4b}$$

Here, we assume that the parametric interactions are based on a third-order nonlinear process, where two pump lights create the signal and idler photons simultaneously. In such systems, the coefficient in the above equations is proportional to the pump light amplitudes of A_{p1} and A_{p2} as $(n/c)\chi \propto A_{p1}A_{p2}$. With the approximation of no pump depletion, the pump amplitude can be expressed as $A_{p1,\,p2}(z)=A_{p1,\,p2}(0)\exp(-i\beta_{p1,\,p2}z)$, and $(n/c)\chi$ can be expressed as $(n/c)\chi=\kappa\exp[-i(\beta_{p1}+\beta_{p2})z]$, where κ is a proportional constant including $A_{p1}(0)$, $A_{p2}(0)$, and the nonlinear coefficient. With these notations, Eqs. (4a) and (4b) are rewritten as

$$\frac{d\hat{b}_s}{dz} = \kappa\hat{b}_i^\dagger e^{i\Delta\beta z}, \tag{5a}$$

$$\frac{d\hat{b}_i^\dagger}{dz} = \kappa^*\hat{b}_s e^{-i\Delta\beta z}, \tag{5b}$$

where $\Delta\beta=\beta_s+\beta_i-\beta_{p1}-\beta_{p2}$ represents the phase mismatch. With the above mentioned process, we obtain the equations of motion of the field operators, which include the phase mismatch $\Delta\beta$, as shown in Eqs. (5a) and (5b). These equations are equivalent to classical coupled-equations derived from nonlinear Maxwell's equations. We derive them from the quantum-mechanical Heisenberg equation in this paper.

It is noteworthy here that $\Delta\beta$ in Eqs. (5a) and (5b) includes the nonlinear phase-shift as well as the linear phase mismatch. In parametric amplification, the pump lights are so strong that the refractive index is shifted due to the self and cross phase modulation effects. The propagation constants in such a situation are expressed as [8]

$$\beta_{s,\,i} = \frac{n_{s,\,i}\omega_{s,\,i}}{c} = \frac{n_{s,\,i}^{(0)} + 2n_2(I_{p1} + I_{p2})}{c}\omega_{s,\,i} = \frac{n_{s,\,i}^{(0)}}{c}\omega_{s,\,i} + 2\gamma(P_{p1} + P_{p2}), \tag{6a}$$

$$\beta_{p1,\,p2} = \frac{n_{p1,\,p2}^{(0)}}{c}\omega_{p1,\,p2} + \gamma(P_{p1,p2} + 2P_{p2,p1}). \tag{6b}$$

where $n^{(0)}$ is the linear refractive index; n_2 is the nonlinear refractive index; I is the light intensity; γ is the nonlinear coefficient; P is the light power; and the signal and idler light powers are assumed to be sufficiently low for not affecting the refractive index. Using these expressions, the phase mismatch $\Delta\beta$ is rewritten as

$$\Delta\beta = \Delta\beta_L + \gamma(P_{p1} + P_{p2}), \tag{7}$$

where $\Delta\beta_L$ is the linear phase mismatch given by

$$\Delta\beta_L = \frac{1}{c}(n_s^{(0)}\omega_s + n_i^{(0)}\omega_i - n_{p1}^{(0)}\omega_{p1} - n_{p2}^{(0)}\omega_{p2}). \tag{8}$$

The phase mismatch can be zero as $\Delta\beta=0$ by appropriately choosing the wavelengths and the incident powers of the signal and pump lights, at which the amplification gain is maximum [4]. However, when the wavelengths and the powers deviate from the optimum values, the phase mismatch becomes non-zero according to Eq. (7). Quantum properties under such conditions can be evaluated by using Eqs. (5a) and (5b) with Eq. (7).

2.2. Phase-insensitive amplification

When the signal and idler lights have different frequencies that satisfy $f_{p1}+f_{p2}=f_s+f_i$ (where $f_{p1,\,p2}$, f_s, and f_i are the pump, signal, and idler frequencies, respectively), and no idler light is incident, signal amplification occurs irrespective of its incident phase (i.e., phase-insensitive amplification). The solution of Eq. (5) for such a system is expressed as

$$\hat{a}_s(\text{out}) = \left\{\cosh(gL) - i(\Delta\beta/2g)\sinh(gL)\right\}\hat{a}_s(\text{in}) + (\kappa/g)\sinh(gL)\hat{a}_i^\dagger(\text{in}), \tag{9}$$

where L is the length of a nonlinear medium and

$$g \equiv \sqrt{|\kappa|^2 - (\Delta\beta/2)^2}. \tag{10}$$

In the Heisenberg picture of quantum mechanics, the expectation value (or mean value) of a physical quantity is given by $<\Psi(\text{in})|\hat{A}(\text{out})|\Psi(\text{in})>$, where $\hat{A}(\text{out})$ is a time-evolved operator corresponding to the physical quantity, and $|\Psi(\text{in})>$ is an initial state of the concerned system. Here, the incident signal light is assumed to be an ideal monochromatic light (i.e., a coherent state in terms of quantum mechanics). The initial state corresponding to such incident condition is expressed as

$$|\Psi(\text{in})> = |\alpha>_s \otimes |0>_i, \tag{11}$$

where $|0>$ and $|\alpha>$ represent the vacuum state and the coherent state of mean amplitude α, respectively. Using Eqs. (9) and (11), the mean amplitude and the mean photon number of the output signal light are obtained as

$$\text{Amplitude: } <\Psi(\text{in})\left|\hat{a}_s(\text{out})\right|\Psi(\text{in})> = \left\{\cosh(gL) - i(\Delta\beta/2g)\sinh(gL)\right\}\alpha, \quad (12a)$$

$$\begin{aligned}\text{Photon number: } &<\Psi(\text{in})\left|\hat{n}_s(\text{out})\right|\Psi(\text{in})> = <\Psi(\text{in})\left|\hat{a}_s^\dagger(\text{out})\hat{a}_s(\text{out})\right|\Psi(\text{in})> \\ &= n_0\left[\cosh^2(gL) + (\Delta\beta/2g)^2\sinh^2(gL)\right] \\ &\quad + \left\{1 + (\Delta\beta/2g)^2\right\}\sinh^2(gL), \end{aligned} \quad (12b)$$

where $\hat{n} \equiv \hat{a}^\dagger\hat{a}$ is the photon number operator, $n_0 \equiv |\alpha|^2$ is the mean photon number of the incident signal light, and $\hat{a}(\text{in})|\alpha> = \alpha|\alpha>$ and $[\hat{a}(\text{in}), \hat{a}^\dagger(\text{in})] = 1$ are utilized. In Eq. (12b), the first and second terms represent the amplified signal photon number and the spontaneously emitted photon number, respectively. The signal gain G is obtained from Eq. (12b) as

$$G = \cosh^2(gL) + (\Delta\beta/2g)^2\sinh^2(gL). \quad (13)$$

This expression equals to the conventional expression of the signal gain in phase-insensitive parametric amplification.

Quantum fluctuations or noises can also be obtained using Eqs. (9) and (11). In general, fluctuation of variable x is evaluated by the variance given by $\sigma^2 = <x^2> - <x>^2$, where $<\ >$ denote the average. The variances of the two quadrature components of the output light field, σ_i^2 and σ_q^2, respectively, and that of the output photon number, σ_n^2, are obtained as

$$\begin{aligned}\sigma_i^2 &= <\Psi(\text{in})\left|\left\{\frac{\hat{a}_s(\text{out}) + \hat{a}_s^\dagger(\text{out})}{2}\right\}^2\right|\Psi(\text{in})> - <\Psi(\text{in})\left|\left\{\frac{\hat{a}_s(\text{out}) + \hat{a}_s^\dagger(\text{out})}{2}\right\}\right|\Psi(\text{in})>^2 \\ &= \frac{1}{4}\left[\cosh^2(gL) + (\Delta\beta/2g)^2\sinh^2(gL) + \left\{1 + (\Delta\beta/2g)^2\right\}\sinh^2(gL)\right], \end{aligned} \quad (14a)$$

$$\begin{aligned}\sigma_q^2 &= <\Psi(\text{in})\left|\left\{\frac{\hat{a}_s(\text{out}) - \hat{a}_s^\dagger(\text{out})}{2i}\right\}^2\right|\Psi(\text{in})> - <\Psi(\text{in})\left|\left\{\frac{\hat{a}_s(\text{out}) - \hat{a}_s^\dagger(\text{out})}{2i}\right\}\right|\Psi(\text{in})>^2 \\ &= \frac{1}{4}\{\cosh^2(gL) + (\Delta\beta/2g)^2\sinh^2(gL) \\ &\quad + \{1 + (\Delta\beta/2g)^2\}\sinh^2(gL)\}, \end{aligned} \quad (14b)$$

$$\begin{aligned}\sigma_n^2 &= <\Psi(\text{in})\left|\hat{n}_s(\text{out})^2\right|\Psi(\text{in})> - <\Psi(\text{in})\left|\hat{n}_s(\text{out})\right|\Psi(\text{in})>^2 \\ &= <\Psi(\text{in})\left|\hat{a}_s^\dagger(\text{out})\hat{a}_s(\text{out})\hat{a}_s^\dagger(\text{out})\hat{a}_s(\text{out})\right|\Psi(\text{in})> \\ &\quad - <\Psi(\text{in})\left|\hat{a}_s^\dagger(\text{out})\hat{a}_s(\text{out})\right|\Psi(\text{in})>^2 \\ &= n_0 G\left\{\cosh(2gL) + 2(\Delta\beta/2g)^2\sinh^2(gL)\right\}, \end{aligned} \quad (14c)$$

where $n_0 \gg 1$ is assumed. These equations indicate quantum noise in phase-insensitive parametric amplification under phase-mismatched conditions.

Using Eq. (14c), NF can be evaluated. It is defined as NF=SNR(in)/SNR(out), where SNR(in) and SNR(out) are the input and output signal-to-noise ratios, respectively, in terms of the photon number (or the light intensity). The output SNR in the photon number is obtained from Eqs. (13) and (14c) as

$$\begin{aligned}\text{SNR(out)} &= \frac{\{n_0 G\}^2}{n_0 G\left\{\cosh(2gL) + 2(\Delta\beta/2g)^2\sinh^2(gL)\right\}} \\ &= n_0\frac{\cosh^2(gL) + (\Delta\beta/2g)^2\sinh^2(gL)}{\cosh(2gL) + 2(\Delta\beta/2g)^2\sinh^2(gL)}. \end{aligned} \quad (15)$$

On the other hand, the SNR of a coherent state is employed as the input SNR in the definition of NF. The mean photon number and its variance of a coherent state are both n_0, and SNR(in)=$\{n_0\}^2/n_0 = n_0$. Thus, the noise figure NF is given by

$$\text{NF} = \frac{\cosh(2gL) + 2(\Delta\beta/2g)^2\sinh^2(gL)}{\cosh^2(gL) + (\Delta\beta/2g)^2\sinh^2(gL)}. \quad (16)$$

When the parametric gain is sufficiently high, i.e., $gL \gg 1$, this expression is approximated as NF ≈ 2 (or 3 dB), which equals to the NF under phase-matched conditions, indicating that the phase mismatch has little effect towards the noise performance in phase-insensitive parametric amplification.

2.3. Phase-sensitive amplification

When the pump and signal frequencies satisfy $f_{p1}+f_{p2}=2f_s$ and the idler light is degenerate with the signal light as $f_i=f_{p1}+f_{p2}-f_s=f_s$, parametric amplification becomes dependent on the signal incident phase (i.e., phase-sensitive amplification). The output of the signal field operator in such a system is obtained from Eq. (5) with $\hat{a}_i = \hat{a}_s$ as

$$\begin{aligned}\hat{a}_s(\text{out}) &= \{\cosh(gL) - i(\Delta\beta/2g)\sinh(gL)\}\hat{a}_s(\text{in}) \\ &\quad + (\kappa/g)\sinh(gL)\hat{a}_s^\dagger(\text{in}), \end{aligned} \quad (17)$$

where $\Delta\beta=2\beta_s-\beta_{p1}-\beta_{p2}$. Here, κ is a proportional constant including $A_{p1,\ p2}(0)$ and the nonlinear coefficient, which can be expressed as $\kappa=|\kappa|\exp[i(\theta_{p1}+\theta_{p2}+\theta_0)]$, where θ_{p1} and θ_{p2} are the initial phases of the two pump lights, respectively, and θ_0 is the constant phase. In addition, $|\kappa|$ satisfies Eq. (10), and κ can be expressed as

$$\kappa = g\sqrt{1 + (\Delta\beta/2g)^2}\,e^{i\varphi}. \quad (18)$$

where $\phi=\theta_{p1}+\theta_{p2}+\theta_0$. By substituting this expression into Eq. (17), we have

$$\begin{aligned}\hat{a}_s(\text{out}) &= \left\{\cosh(gL) - i(\Delta\beta/2g)\sinh(gL)\right\}\hat{a}(\text{in}) \\ &\quad + \sqrt{1 + (\Delta\beta/2g)^2}\,e^{i\varphi}\sinh(gL)\hat{a}^\dagger(\text{in}). \end{aligned} \quad (19)$$

Using Eq. (19) and the initial state expressed by Eq. (11), the mean amplitude and the mean photon number of the signal output light in phase-sensitive amplification are obtained as follows:

$$\begin{aligned}\text{Amplitude: } <\Psi(\text{in})\left|\hat{a}_s(\text{out})\right|\Psi(\text{in})> &= \sqrt{n_0}\Big[\{\cosh(gL) - i(\Delta\beta/2g)\sinh(gL)\}e^{i\theta_s} \\ &\quad + \sqrt{1 + (\Delta\beta/2g)^2}\sinh(gL)e^{-i(\theta_s-\varphi)}\Big]. \end{aligned} \quad (20a)$$

$$\text{Photon number: } \langle \Psi(\text{in}) | \hat{n}_s(\text{out}) | \Psi(\text{in}) \rangle = \langle \Psi(\text{in}) | \hat{a}_s^\dagger(\text{out}) \hat{a}_s(\text{out}) | \Psi(\text{in}) \rangle$$

$$= n_0 \Big[\cosh(2gL) + 2(\Delta\beta/2g)^2 \sinh^2(gL) + 2\sqrt{1+(\Delta\beta/2g)^2} \Big\{ \frac{\sinh(2gL)}{2} \cos(2\theta_s - \varphi) + (\Delta\beta/2g)\sinh^2(gL)\sin(2\theta_s - \varphi) \Big\} \Big] + \Big\{ 1 + (\Delta\beta/2g)^2 \Big\} \sinh^2(gL), \quad (20b)$$

where the signal incident amplitude is expressed as $\alpha = \sqrt{n_0}\exp(i\theta_s)$ with θ_s being the signal incident phase. The first and second terms in Eq. (20b) represent the amplified signal photon number and the spontaneously emitted photon number, respectively. The phase-sensitive property is indicated in the above equations. From Eq. (20b), the signal gain is obtained as

$$G = \cosh(2gL) + 2(\Delta\beta/2g)^2 \sinh^2(gL) + 2\sqrt{1+(\Delta\beta/2g)^2} \Big\{ \frac{\sinh(2gL)}{2} \cos(2\theta_s - \varphi) + (\Delta\beta/2g)\sinh^2(gL)\sin(2\theta_s - \varphi) \Big\}. \quad (21)$$

When the phase synchronization between the pump and signal is made as $2\theta_s = \phi$, Eq. (21) becomes

$$G = \cosh(2gL) + 2(\Delta\beta/2g)^2 \sinh^2(gL) + \sqrt{1+(\Delta\beta/2g)^2}\,\sinh(2gL). \quad (22)$$

Using Eqs. (11) and (19), the variances of the two quadrature components of the light field and the photon number, under the phase-synchronized condition of $2\theta_s = \phi$, are also obtained by the same procedure in the previous subsection as

$$\sigma_i^2 = \frac{1}{4}\Big[\cosh(2gL) + \sqrt{1+(\Delta\beta/2g)^2}\,\sinh(2gL) + 2(\Delta\beta/2g)^2 \sinh^2(gL) \Big], \quad (23a)$$

$$\sigma_q^2 = \frac{1}{4}\Big[\cosh(2gL) - \sqrt{1+(\Delta\beta/2g)^2}\,\sinh(2gL) + 2(\Delta\beta/2g)^2 \sinh^2(gL) \Big], \quad (23b)$$

$$\sigma_n^2 = n_0\Big[G^2 + 4(\Delta\beta/2g)^2 \Big\{ 1 + (\Delta\beta/2g)^2 \Big\} \sinh^4(gL) \Big] \quad (23c)$$

where $n_0 \gg 1$ is assumed.

Using Eqs. (22) and (23c), NF=SNR(in)/SNR(out) in the photon number is obtained as

$$\text{NF} = \frac{n_0}{\{n_0 G\}^2/\sigma_n^2} = 1 + \frac{4}{G^2}(\Delta\beta/2g)^2\{1 + (\Delta\beta/2g)^2\}\sinh^4(gL). \quad (24)$$

When the parametric gain is sufficiently high, this expression is approximated as

$$\text{NF} \approx 1 + \frac{e^{2gL}}{2} \cdot \frac{(\Delta\beta/2g)^2\sqrt{1+(\Delta\beta/2g)^2}}{1+\sqrt{1+(\Delta\beta/2g)^2}}. \quad (25)$$

Eq. (25) suggests that NF in phase-sensitive amplification definitely depends on the phase mismatch $\Delta\beta$, in contrast to phase-insensitive amplification.

3. Calculation

We carried out calculations based on the above results. Fig. 1 shows NF and signal gain G as a function of phase mismatch $\Delta\beta$ in phase-insensitive amplification. NF is almost constant, independent of the phase mismatch under high-gain and medium-gain conditions, whereas it decreases slightly as $\Delta\beta$ increases under a low-gain condition.

Fig. 2 shows NF and signal gain as a function of $\Delta\beta$ in phase-sensitive amplification. Unlike phase-insensitive amplification, NF notably depends on $\Delta\beta$, especially under a high-gain condition, such that it increases as $\Delta\beta$ increases. This result indicates that it is important for the phase-matching condition to be satisfied in order to obtain a low-noise property in phase-sensitive amplifiers.

4. Discussion

The results obtained above, that the noise figure is notably dependent on the phase mismatch in phase-sensitive amplification (PSA) while it is not in phase-insensitive amplification (PIA), are understood as follows.

The noise performance of an amplifier can be indicated by the spontaneously emitted photon number (which is the source of quantum noise) relative to the signal gain, i.e., n_{ASE}/G with n_{ASE} being the photon number of amplified spontaneous emission (ASE) and G being the signal gain. From Eqs. (12b) and (20b), the ASE photon number and the signal gain in parametric amplification are given by

$$\text{PIA: } n_{ASE}^{(PI)} = \Big\{ 1 + (\Delta\beta/2g)^2 \Big\} \sinh^2(gL), \quad (26a)$$

$$G_{PI} = \cosh^2(gL) + (\Delta\beta/2g)^2 \sinh^2(gL), \quad (26b)$$

$$\text{PSA: } n_{ASE}^{(PS)} = \Big\{ 1 + (\Delta\beta/2g)^2 \Big\} \sinh^2(gL), \quad (27a)$$

$$G_{PS} = \cosh(2gL) + 2(\Delta\beta/2g)^2 \sinh^2(gL) + \sqrt{1+(\Delta\beta/2g)^2}\,\sinh(2gL), \quad (27b)$$

where the pump powers and the medium length are assumed to be identical in PIA and PSA, and the phase synchronization is assumed to be made in PSA. The above equations show that, under these conditions, the ASE photon number is the same in PIA and PSA while the signal gain is different.

When the phase matching condition is satisfied, i.e., $\Delta\beta = 0$, and the signal gain is large, i.e., $gL \gg 1$, the signal gains is $G_{PI} = \cosh^2(gL) \approx e^{2gL}/4$ for PIA and $G_{PS} = \cosh(2gL) + \sinh(2gL) \approx e^{2gL}$ for PSA, that is, the latter is four times the former under the condition that the ASE photon number is identical. In other words, the ASE photon number in PSA is one fourth of that in PIA, provided that the signal gain is equal. This consideration concludes that PSA has better noise performance than PIA. Note here that ASE light amplitude is squeezed along the signal phase direction in PSA while that in PIA has uniformly distributed phases. Thus, whole ASE lights contribute to signal-spontaneous beat noise in PSA while a half of them does so in PIA. Then, with the condition that the ASE photon number in PSA is one fourth of that in PIA for an identical gain, PSA has a 3-dB better NF than PIA, which is a well-known noise property in parametric amplification.

The above gain characteristic, i.e., PSA has four time higher gain than PIA for identical pump powers and the medium length, is due to the fact that signal and idler lights in parametric amplification are degenerate in PSA. In Eq. (20a) that shows the PSA output amplitude, the first term of $\{\cosh(gL) - i(\Delta\beta/2g)\sinh(gL)\}\exp(i\theta_s)$ corresponds to the signal component, and the second term of $\{1 + (\Delta\beta/2g)^2\}^{1/2}\sinh(gL)\exp[-i(\theta_s - \phi)]$ corresponds to the idler one. When the phase matching condition is satisfied and the signal gain is high, they are $\cosh(gL)\exp(i\theta_s) \approx (e^{gL}/2)\exp(i\theta_s)$ and $\sinh(gL)$

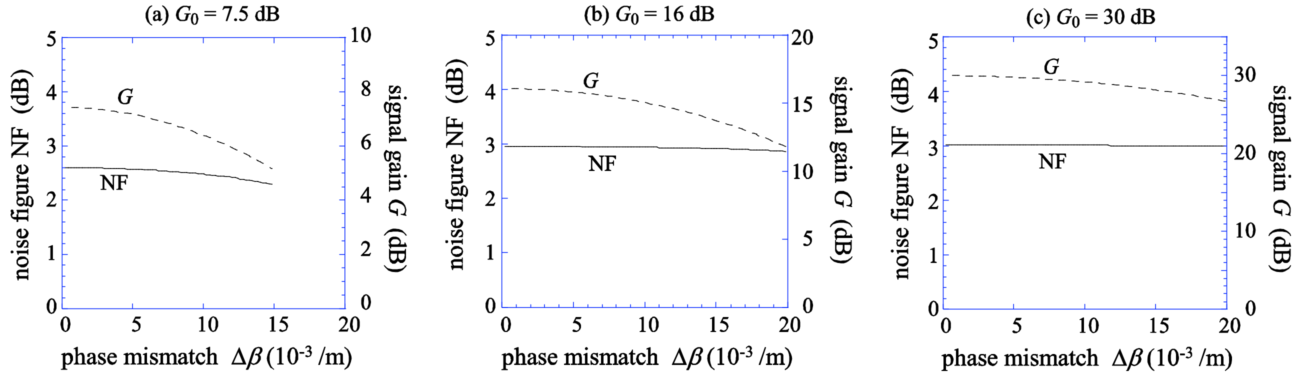

Fig. 1. Signal gain and the NF as a function of phase mismatch $\Delta\beta$ in phase-insensitive amplification. The gain coefficient g and medium length L are chosen in order that the gain at phase-matched condition G_0 is 7.5 dB (a), 16 dB (b), and 30 dB (c). Calculations are performed over a range of $\Delta\beta$ in which g defined in Eq. (10) is positive.

$\exp[-i(\theta_s-\phi)] \approx (e^{gL}/2)\exp[-i(\theta_s-\phi)]$, respectively, indicating that the two components have a nearly equal amplitude under these conditions. When the phase synchronization is made in addition, they have an identical phase. Thus, the two components with an equal amplitude is summed in phase, and then the total power is four times the signal component alone. In PIA, on the other hand, the output light includes only the signal component, as shown in (12a). Therefore, the PSA gain is four times the PIA gain when the pump powers and the medium length are identical.

The above consideration is for the phase-matched condition. When phase mismatch occurs as $\Delta\beta\neq 0$, the ASE photon number and the signal gain under high gain conditions become

$$\text{PIA: } n_{ASE}^{(PI)} \approx \left\{1+(\Delta\beta/2g)^2\right\}e^{2gL}/4, \tag{28a}$$

$$G_{PI} \approx \left\{1+(\Delta\beta/2g)^2\right\}e^{2gL}/4, \tag{28b}$$

$$\text{PSA: } n_{ASE}^{(PS)} \approx \left\{1+(\Delta\beta/2g)^2\right\}e^{2gL}/4, \tag{29a}$$

$$G_{PS} \approx \left\{1+(\Delta\beta/2g)^2+\sqrt{1+(\Delta\beta/2g)^2}\right\}e^{2gL}/2, \tag{29b}$$

and their ratio is

$$\text{PIA: } \frac{n_{ASE}^{(PI)}}{G_{PI}} \approx 1, \tag{30}$$

$$\begin{aligned}\text{PSA: } \frac{n_{ASE}^{(PS)}}{G_{PS}} &\approx \frac{1}{2}\cdot\frac{1}{1+1/\sqrt{1+(\Delta\beta/2g)^2}},\\ &= \frac{1}{2}\cdot\frac{1}{1+1/\sqrt{1+\Delta\beta^2/4(|\kappa|^2-(\Delta\beta/2)^2)}}\\ &= \frac{1}{2}\cdot\frac{1}{1+\sqrt{1-\Delta\beta^2/4|\kappa|^2}},\end{aligned} \tag{31}$$

where Eq. (10) is substituted. The above equations indicate that, with $|\Delta\beta|$ increasing from 0, the ASE photon number and the signal gain decrease in a same manner in PIA, while the signal gain rapidly decreases compared with the ASE photon number in PSA. Therefore, the noise performance scarcely changes with the phase mismatch in PIA while it notably degrades in PSA, as shown in Figs. 1 and 2.

The rapid decrease of the PSA gain with the phase mismatch is due to phase detuning between the signal and idler components. Under the phase-matched condition, the single and idler components are summed in phase, as described above. When the phase mismatch occurs, the signal and idler phases are detuned, and they are not summed in phase anymore, resulting in the gain reduction. Thus, the PSA gain reduction is larger than the ASE reduction, as the phase mismatch increases. This is an intuitive explanation for the NF degradation shown in Fig. 2.

This paper has mainly considered noise properties in terms of photon number or light intensity, because the amplifier noise performance has been traditionally evaluated by NF in intensity. In the last part of the paper, we briefly mention about amplitude noise. Its dependence on the phase mismatch is similar to intensity noise, such that the output signal-to-noise ratio scarcely depends on the phase mismatch in PIA while it notably does in PSA. An additional property in PSA is that the squeezing ratio of

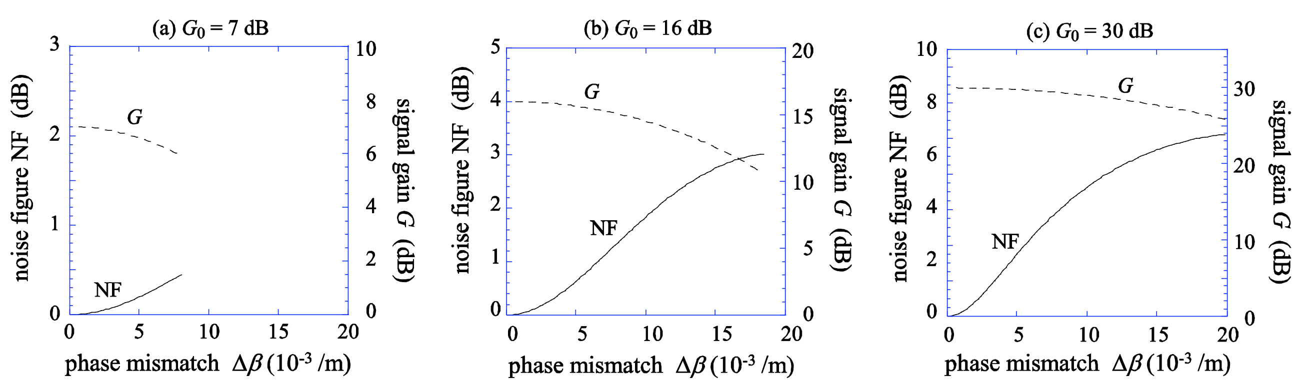

Fig. 2. Signal gain and noise figure as a function of phase mismatch $\Delta\beta$ in phase-sensitive amplification. The gain coefficient g and medium length L are chosen such that the gain at the phase matched condition G_0 is 7 dB (a), 16 dB (b), and 30 dB (c). Calculations are performed over a range of $\Delta\beta$ in which g defined in Eq. (10) is positive.

amplitude noise also depends on the phase mismatch. Eqs. (23a) and (23b) indicate that the variances of the two quadrature components are different such that the i-component has a larger variance than the q-component, i.e., the amplitude noise is squeezed along the i-axis. Under a high-gain condition of $gL \gg 1$, these variances are approximated as

$$\sigma_{\mathrm{i}}^2 \approx \frac{e^{2gL}}{8}\left[1 + (\Delta\beta/2g)^2 + \sqrt{1 + (\Delta\beta/2g)^2}\right], \tag{32a}$$

$$\sigma_{\mathrm{q}}^2 \approx \frac{e^{2gL}}{8}\left[1 + (\Delta\beta/2g)^2 - \sqrt{1 + (\Delta\beta/2g)^2}\right]. \tag{32b}$$

Then, the squeezing ratio is evaluated as

$$\frac{\sigma_{\mathrm{i}}^2}{\sigma_{\mathrm{q}}^2} \approx \frac{1 + (\Delta\beta/2g)^2 + \sqrt{1 + (\Delta\beta/2g)^2}}{1 + (\Delta\beta/2g)^2 - \sqrt{1 + (\Delta\beta/2g)^2}} = \frac{1 + \sqrt{1 - \Delta\beta^2/4|\kappa|^2}}{1 - \sqrt{1 - \Delta\beta^2/4|\kappa|^2}}, \tag{33}$$

where Eq. (10) is substituted. This equation indicates that, as the phase mismatch increases, the squeezing ratio reduces, and the amplitude noise distribution approaches to be isotropic.

5. Summary

We studied quantum noise in parametric amplification under phase-mismatched conditions. Equations of motion of the field operators including the phase mismatch were derived from the Heisenberg equation, and then, NF was calculated using the solutions of the derived equations. The results showed that phase mismatch $\Delta\beta$ has little effect in phase-insensitive amplification, while $\Delta\beta$ has notable effect in phase-sensitive amplification.

References

[1] J.A. Levenson, I. Abram, Th Rivera, Ph Grangier, J. Opt. Soc. Am. B 10 (1993) 2233–2238.
[2] J.A. Levenson, K. Bencheikh, D.J. Lovering, P. Vidakovic, C. Simonneau, Quantum Semiclass. Opt. 9 (1997) 221–237.
[3] C.J. McKinstric, M.G. Raymer, S. Radic, M.V. Vasilyev, Opt. Commun. 257 (2006) 146–163.
[4] M.E. Marhic, Fiber Optical Parametric Amplifiers, Oscillators and Related Devices, Cambridge University Press, New York, 2008.
[5] Z. Tong, C.J. McKinstrie, C. Lundström, M. Karlsson, P.A. Andrekson, Opt. Express 18 (2010) 15426–15439.
[6] C.M. Caves, J. Combes, Z. Jiang, S. Pandey, Phys. Rev. A 86 (2012) 063802.
[7] E.F. Walls, G.J. Milburn, Quantum Optics, 2nd ed., Springer, Berlin, 2007.
[8] G.P. Agrawal, Nonlinear Fiber Optics, 3rd ed., Academic Press, San Diego, 2001.